Improving BDDs Manipulation Through Incremental Reduction and Enhanced Heuristics

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Summary

- Introduction
- MBDs and Boolean Verification
- Ordering of Input Variables
- Incremental Manipulation
- Benchmark Results
- Conclusions and Future Work

Canonical BDDs Characteristics

- Used to represent and manipulate Boolean functions (general);
- Applied to design verification, symbolic simulation, logic synthesis, etc;
- Subject to a problem: Establishing the variable ordering.

Our Approach:

- Generalization of BDDs for logic synthesis applications;
- New initial ordering heuristics;
- Incremental techniques to change the ordering dynamically.

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Example

f

x_0

0	0	X	X
0	X	1	X

x_1 x_1

x_2

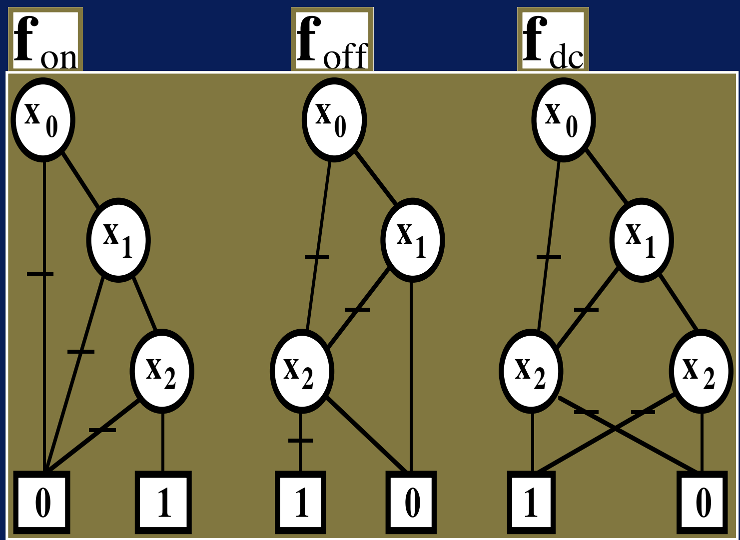
g

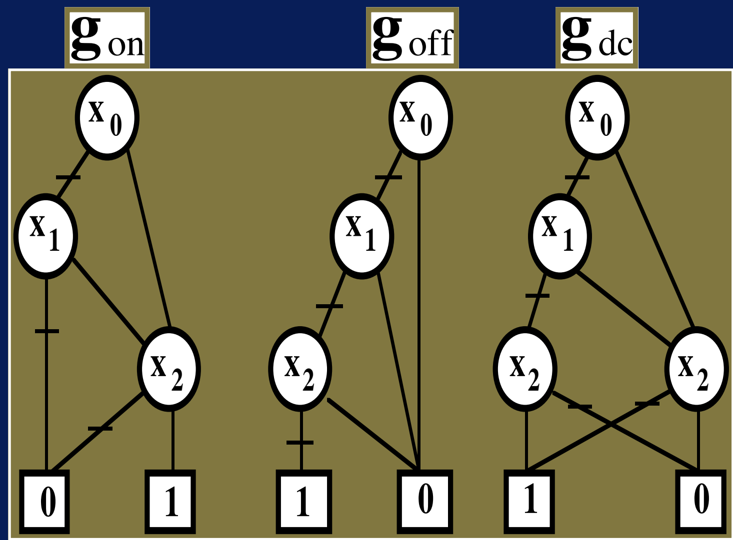
x_0

0	X	1	X
X	X	1	1

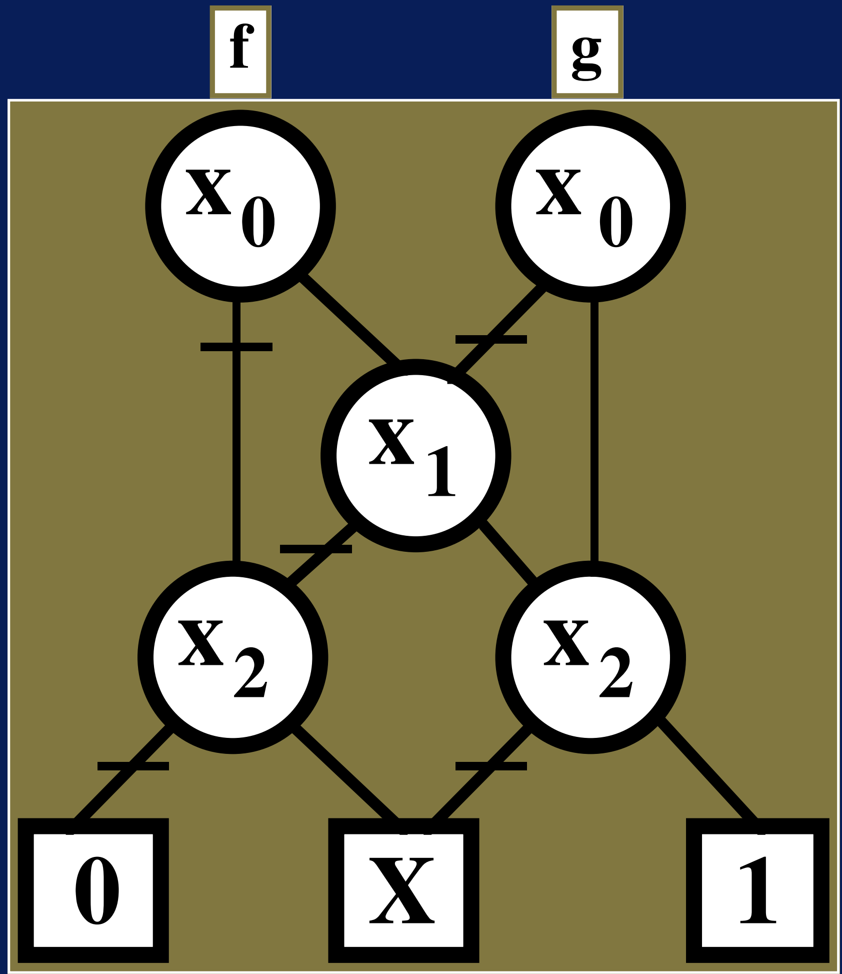
x_1 x_1

x_2





Corresponding MBD



Boolean Verification

Relies on the computation of:

$$\left[g_{on} \subset \left[f_{on} + f_{dc}\right]\right] * \left[f_{on} \subset \left[g_{on} + g_{dc}\right]\right]$$

If this results in a tautology, f≡g.

- **For BDDs, 2-4 calls to** apply**;**
- **For MBDs, 1-2 calls to** apply**;**
- **For CSFs, no calls to** apply **needed.**

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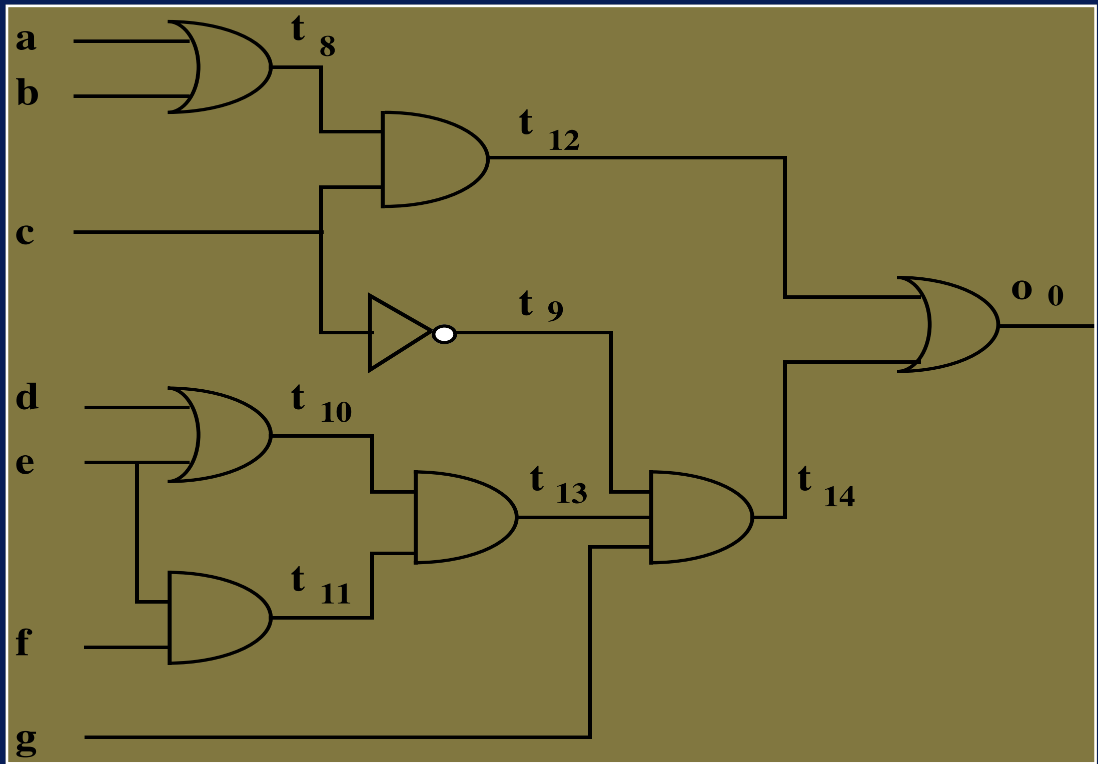
Initial Ordering for Input Variables

- Exact solution to date: $O(n^2 3^n)$;
- Heuristics are unavoidable;
- Fujita et al. proposed depth-first search with pivots.

Depth-first ordering example



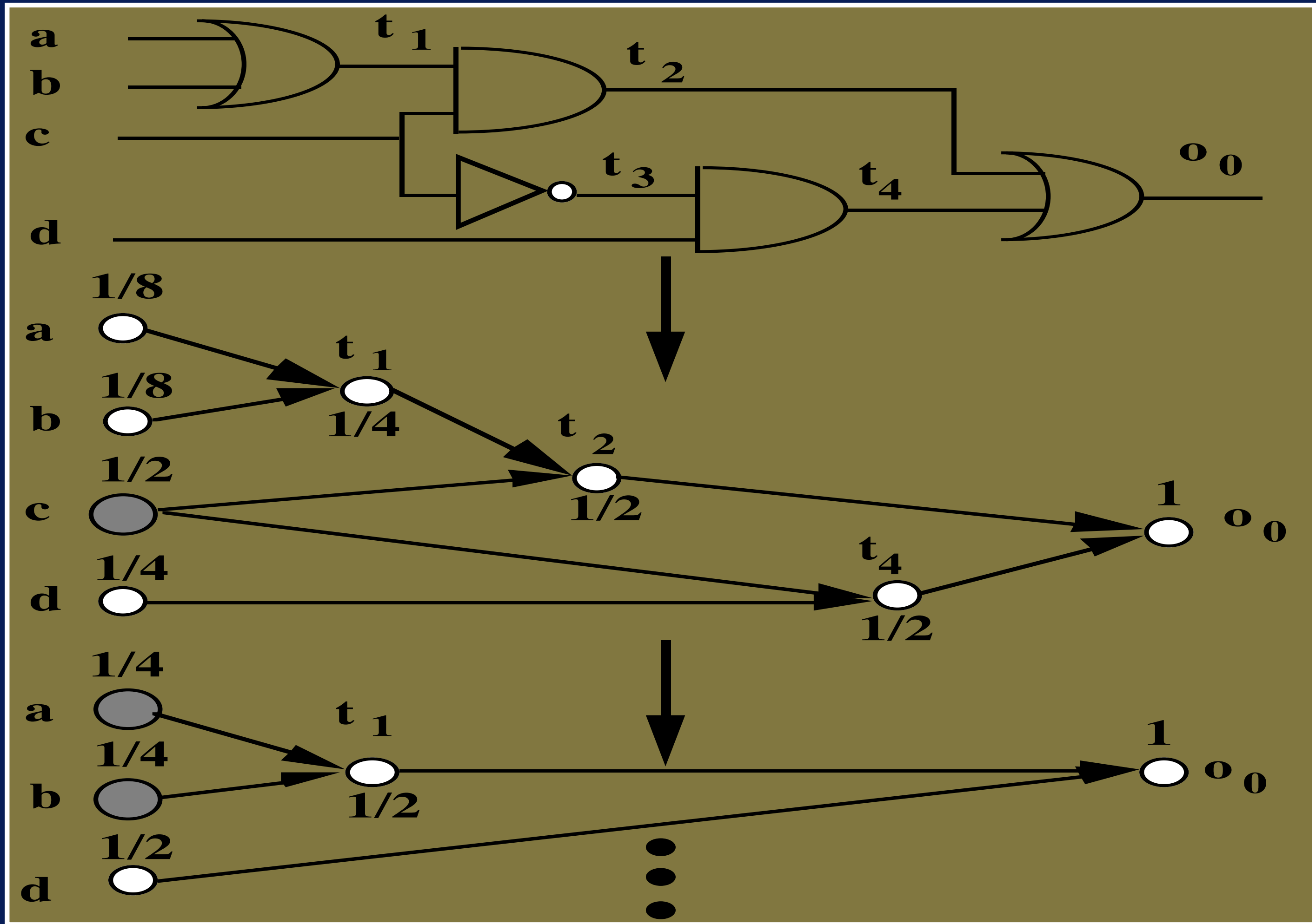
- One possibility: <(c b a) (e d g f)>

Tentative Enhancements

- Sort intermediate/output variables;
- Sort pivoted lists using transitive fan-in;
- Combination of both.

"Results were still rather erratic and dependent on input description."

Weighted Nodes Heuristic



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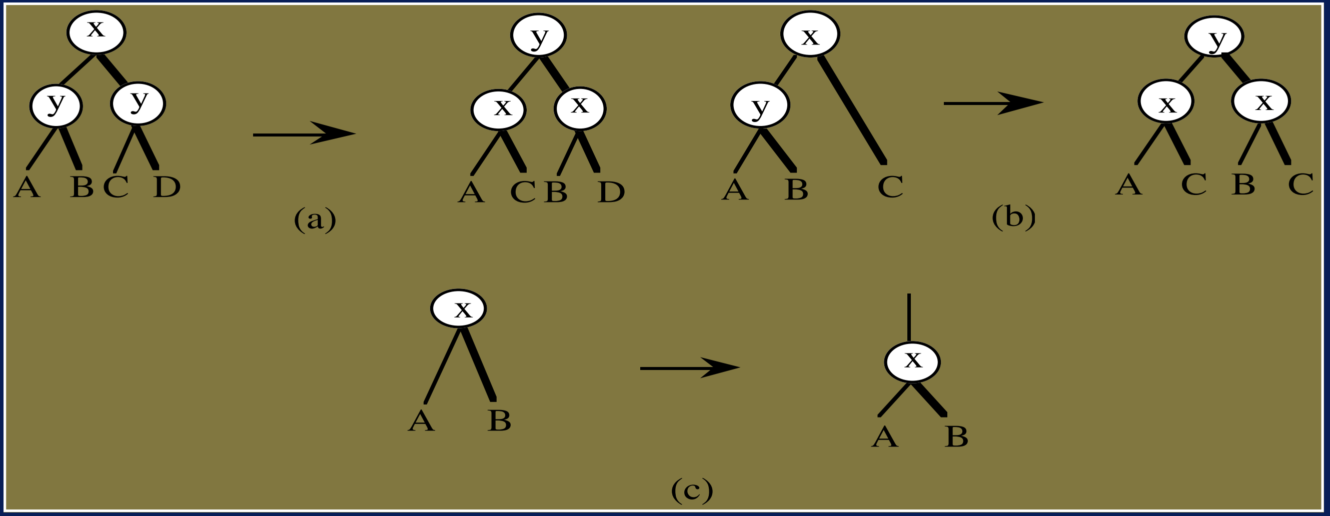
Supportive Statements

- "Exchanging two adjacent variables in the ordering underlying a BDD changes only the levels of the BDD involved in the operation."

- "The reduced BDD corresponding to the new ordering differs from the original reduced BDD only in the exchanged levels."

Operations needed

- ***Swap***, in order to exchange two variables in the ordering;



- ***Local-Reduce***, in order to put the BDD back into a canonical form.

Example of Application:

Incremental reduction of the number of nodes in an MBD using heuristics

- **Successive applications of *Swap* + *Local-Reduce* on selected pairs of adjacent levels ;**
- **Heuristics for ordering selection of pairs and stop conditions:**
  - **best-pair swap (*swap-all-red*);**
  - **greedy swap (*swap-run-down*).**
- **Best results were obtained with sequential application of both heuristics.**

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In our benchmarks:

- MBDs provided average gain of 35% over separated BDDs;
- Initial MBDs are 10% smaller in average, if weighted nodes heuristics is used;

Difference is the same after incremental reduction;

- Incremental reduction provided additional average gain of 21%.

- More than 40 examples of various sizes run. Most benchmarks taken from MCNC and ISCAS.

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Conclusions

- MBDs - a compact and efficient way of representing Boolean functions;
- Underlying structure of network is explicit in MBDs;
- Sharing among functions is also explicit;
- Single graph for the whole network, on-, off-, and dc-sets;
- New heuristic + inc. reduction = smaller MBDs, thus faster execution;
- Incremental techniques provide a way to surpass the intrinsic limitation of BDDs/MBDs (total ordering of input variables).

Future Work

- Extraction of sum-of-products representation from MBDs;
- Factorization and decomposition;
- Boolean division and other optimization techniques;
- Sequential circuits considerations.